Afr. J. Comp. & ICT
Vol. 1, No. 1, 2008, pp. 33 - 45
ISSN 2006-1781

IMPLEMENTATION OF COMPUTATIONALLY EFFICIENT ALGORITHMS FOR MULTIRATE DIGITAL SIGNAL PROCESSING SYSTEMS

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Received 08 May, 2008

Communicated by Oliver Osuagwu

ABSTRACT

Digital Signal Processing (DSP) has become one of the most powerful techniques in reshaping science and engineering in the areas of communication, medical imaging, radar, hi-fi music reproduction, oil prospecting etc. In this paper, Multirate DSP where the signal at a given sampling rate needs to be converted into another signal with a different sampling rate are investigated. Noble identities and polyphase decomposition of linear filters which are computationally more efficient approaches are illustrated. The results showed that the number of filter operations as well as the number of memory required were reduced by a factor of M and L (where M and L are decimation and interpolation factors respectively). The use of this technique promises cheaper DSP hardware that dissipates less heat.

Keywords: Multirate Digital Signal Processing, Digital Filters, Up-sampling, Down-sampling, Decimators, Interpolators and Polyphase realization of FIR filters.

I.0 INTRODUCTION

Presently, there are situations when there is a need to change the sampling frequency of a digitized signal. Some of the reasons for re-sampling may be when there is a need to pass data between two systems that use incompatible sampling rates e.g. transferring music from Digital Audio Tape (DAT) to Compact Disc (CD) or when mixing signals with different standards for example, CD players, digital audio tapes and digital broadcasting all having different sampling frequencies: 44.1KHz for CDs, 48KHz for DATs, 32KHz for digital broadcasting[1, 2]. Multirate DSP is also applied in GSM receivers where the sampling rate conversion (from high to low) has to be made before the signal is processed further for decoding, error detection, correction and equalization.

Multrate DSP operations: Decimation, Interpolation and Fractional (Rational) Sampling Rate Conversion are reviewed in section 2 of this paper. The key to a more efficient implementation of digital filters which are constituents of Multirate systems viz: the Noble Identities and Polyphase Decomposition of linear filters are discussed in Section 3. In section 4, a computationally more efficient algorithm for Multirate DSP in which both the number of filter operations and the amount of memory required are reduced by the decimation and interpolation factors are developed and simulated with Mathlab. The conclusions and recommendations for further studies are in Section 5.

II.0 MULTIRATE DSP OPERATIONS

Whenever a signal at one rate has to be used by a system that expects a different rate, the re-sampling and some processing are required. In view of the fact that the sampling rate affects the re-constructability of the original signal (Nyquist sampling criterion), the- main operations of Multirate DSP systems: Decimation, Interpolation and Fractional (Rational) Sampling Rate Conversion are reviewed in the following sections.

II.1 Decimation

The reduction of a sampling rate is called decimation, because the original sample set is reduced (decimated). Decimation consists of two stages: filtering and down sampling, as shown in Fig. 2.1. Down-sampling reduces the input sampling rate, f_1 by an integer factor M, which is known as a down-sampling factor. The output signal y [m] is called a down-sampled signal and is obtained by taking only every M-th sample of the input signal and discarding all others [3, 4, 5].

$$y(n) = \begin{cases} x(n/L) & for\ n = mL \\ 0 & Otherwise \end{cases} ;$$

m = …, -1, 0, 1,… (2.2)

Y[m]=x[mM]…...(2.1)

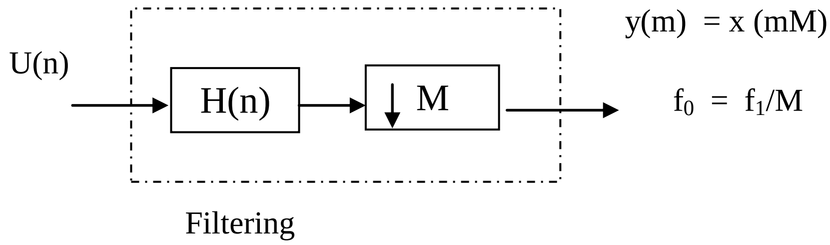


Fig 2.1: Decimation

II.2 Interpolation

The process of increasing sampling rate of a digital signal is called interpolation, and it consists of two stages: Up-sampling and filtering as shown in Fig. 2.2.

II.3 Rational Sampling Rate Conversion

Changing the sampling rate by a ratio of two integers, L/M can be performed by cascading decimation by a factor M and interpolation by a factor L.

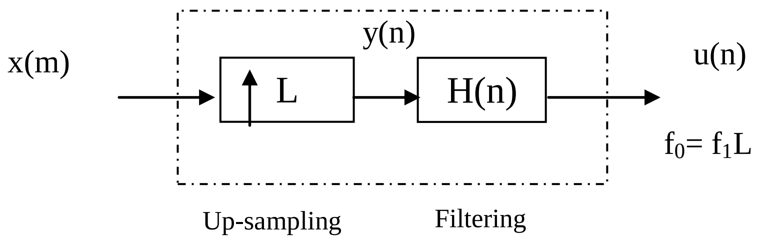


Fig 2.2: Interpolation.

Up-sampling increases the sampling rate by an integer factor L, this involves inserting L – 1 equally spaced zeros between each pair of samples [5, 6].

There are two possible cascade connections depending on what is performed first, decimation or interpolation [3]. The two filters ($H_1(z)$ and $H_2(z)$) can be combined into one filter G(z), as shown in Fig. 2.3, to yield the desired structure for the rational sampling rate conversion.

efficient implementation. These concepts are illustrated in the following section.

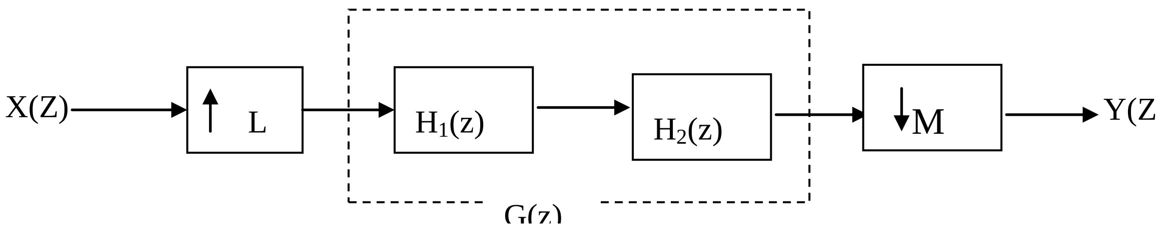


Fig 2.3: Rational sampling

III.0 INEFFICIENCIES IN DIRECT IMPLEMENTATION

In decimation and interpolation, there are two situations: either a linear filter followed by a down-sampler, or an up-sampler followed by a linear filter respectively (Figs. 2.1 and 2.2).

In the case of a decimator by a factor M, for every M sample at the output of the linear filter, only one is retained by the down-sampler and the other M – 1's are discarded. Therefore, they computed for nothing. The same applies to an interpolator by a factor L. At the input of the filter, L -1 out of L samples are zero, and only one is a useful data point. Consequently, there is a need to implement the filters in a more efficient manner to avoid the unnecessary computations. From the study carried out, it was found that the use of the concept of the noble (useful) identities, and the polyphase decomposition of linear filters gives a more

III.1 Noble Identities For Polyphase Decimation

The first identity states that the sum of the scaled, individually down-sampled signals is the same as the down-sampled sum of these signals. This property follows directly from the principle of superposition as illustrated in Fig. 3.1.

The second identity (Fig. 3.2) establishes that a delay of M samples before the down-sampler is equivalent to a delay of one sample after the down-sampler.

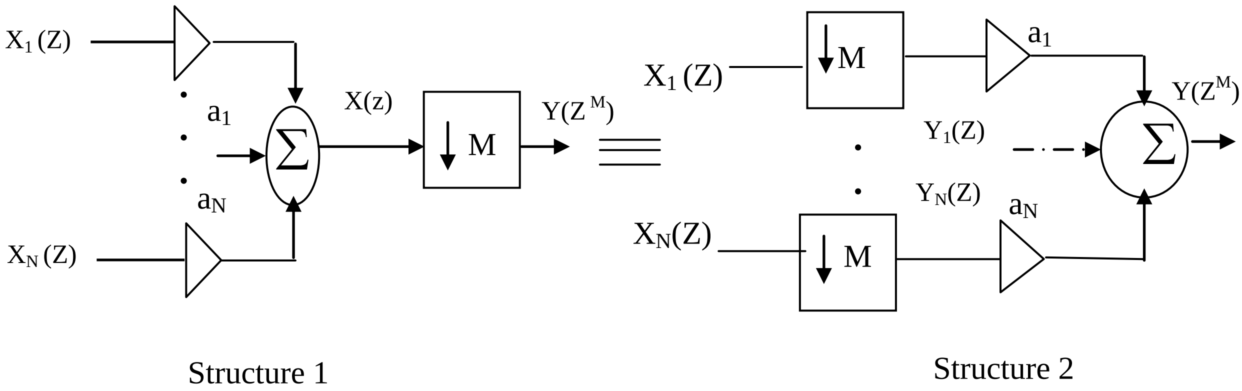


Fig 3.1: Superposition Principle.

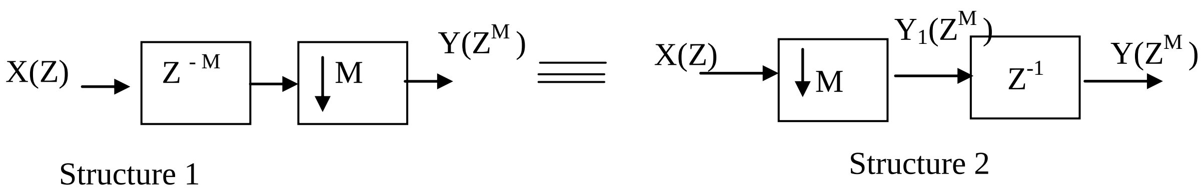


Fig 3.2 : Second Identity

Fig. 3.3 demonstrates the third identity. The filter $G(z^M)$ is called an expanded filter and is obtained by replacing each delay z^{-1} of the original filter G (z) with a delay z^{-M}. In the time domain, this is equivalent to inserting M – 1 zeros between the original samples of the impulse response.

This third identity states that filtering by the expanded filter followed by down-sampling is equivalent to having down-sampling first, followed by the filtering with the original filter. The algebraic proof of these identities is illustrated in [3].

III.2 Noble Identities For Polyphase Interpolation

The three useful identities of the down-sampled signals already stated also apply to up sampling:

Hence the fourth identity, is the superposition principle, i.e; the output signals Y_1 (z), ….Y_N (z), which are obtained by up-sampling the input signal and then scaling by $a_1, \ldots, a_N$, respectively give the same result as if the signal is first scaled and then up-sampled. The fifth identity states that a delay of one sample before up-sampling is equivalent to the delay of L samples after up-sampling. The sixth identity, which is a more general version of the fifth identity states that filtering followed by up-sampling is equivalent to having up-sampling first followed by expanded filtering.

an FIR filter is based on the Polyhase decomposition of its transfer function and

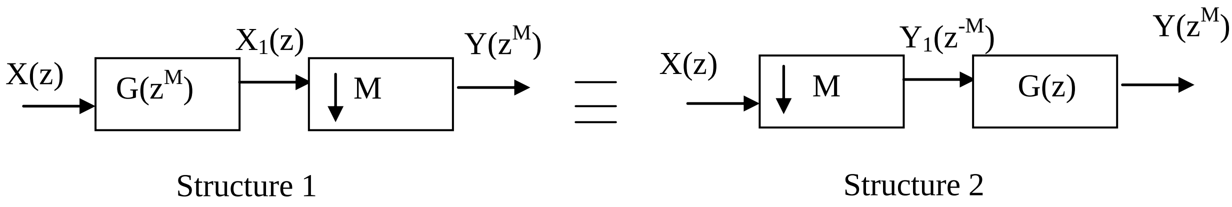


Fig 3.3 : Third identity

III.3 Polyphase Realization Of FIR Filters

The transfer function, H(Z) of a causal FIR filter of order N is characterized by

$$H(z) = \sum_{K=0}^{N} h[k] z^{-k}$$

……………………………...(3.1)

which is a polynomial in z^{-1} of degree N.

In the time domain, the input – output relation of the above FIR filter is given by

$$y[n] = \sum_{K=0}^{N} h[k] x[n-k]$$

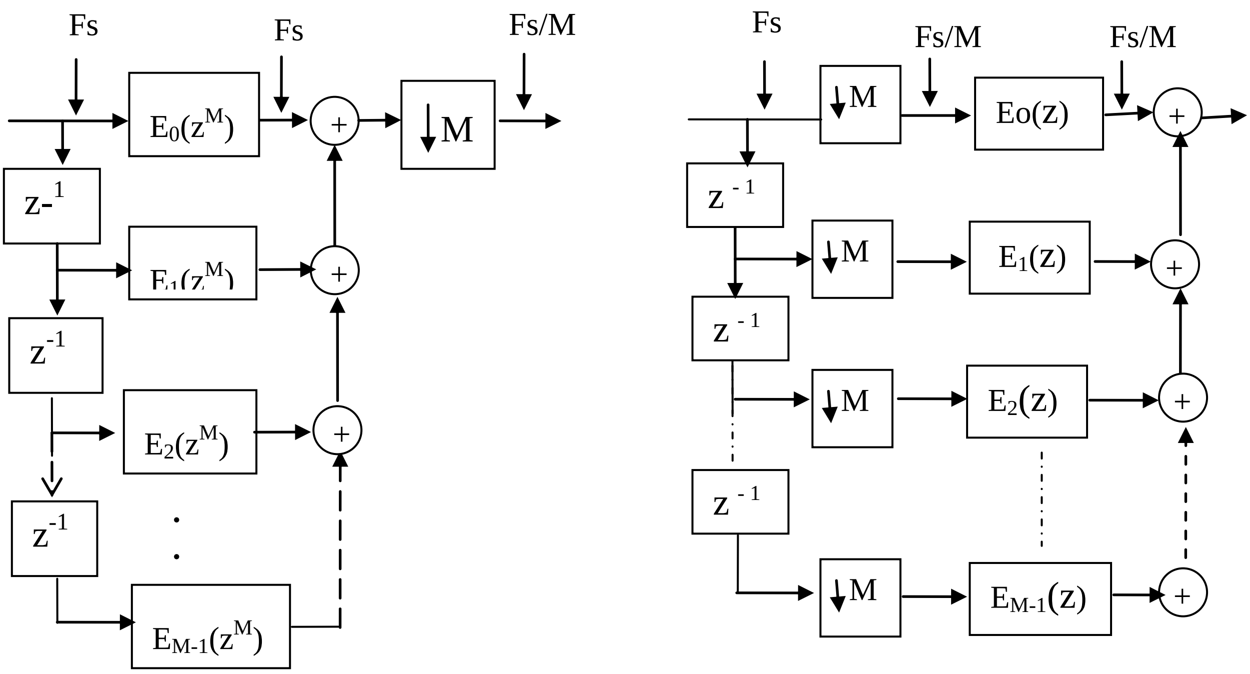
……………………………...(3.2)

where y[n] and x [n] are the output and input sequences, respectively. This realization of

An FIR filter is based on the Polyhase decomposition of its transfer function and results in *a parallel structure* which is more efficient than a serial structure[1, 3].

III.4 Computationally Efficient Decimator Structures

Computationally efficient decimator structures employing low-pass filters can be derived by applying a polyphase decomposition to their corresponding transfer functions. Consider the use of the polyphase decomposition in the realization of the decimation filter of figure 2.1. The overall decimator structure is of the form of figure 3.4(a) when the lowpass filter H(z) is realized using polyphase decomposition. An equivalent realization in figure 3.4(b) is computationally more efficient than the structure figure 3.4(a), obtained using the second noble identity for polyphase decimation (which establishes that a delay of M samples before the down-sampler is equivalent to a delay of one sample after the down-sampler). more efficient version in which both the number of filter operations as well as the amount of memory required are reduced by a factor M is realized. Thus filtering is now performed at a lower sampling rate.



(a) (b)

Fig 3.4: (a) Decimator implementation based on polyphase decomposition.
(b):Computationally efficient decimator structure.

In the figures, the sampling frequencies of pertinent sequences are indicated by an arrow. As can be seen from the figures, a

III.5 Polyphase Interpolation

In the interpolation structure of fig 2.2, the filtering is performed at the higher sampling rate. That means that most of the data values going into the filter are zero, more precisely L – 1 out of every L value. As a consequence in the process of convolution there are many unnecessary multiplications with zero. The convolution at the higher sampling rate can be replaced by independent convolutions at the lower input sampling rate using polyphase decomposition (Eq.3.3). The coefficient M can be replaced by L to obtain.

$$H(z) = \sum_{K=0}^{L-1} z^{-K} \left(z^{L}\right) \quad \text{……………………………(3.3)}$$

Similar savings are also obtained in the case of the interpolator structure employing polyphase decomposition in the realization of computationally efficient interpolators.

Figure 3.5 shows the interpolator structure derived from figure 2.2 by making use of the L – branch Polyphase decomposition of the interpolation filter H(z) and the sixth identity for polyphase interpolation (filtering followed by up-sampling is equivalent to having up-sampling first followed by expanded filtering). Since filtering is now performed at a lower sampling rate, the number of filter operations as well as the amount of memory required will be reduced by a factor L.

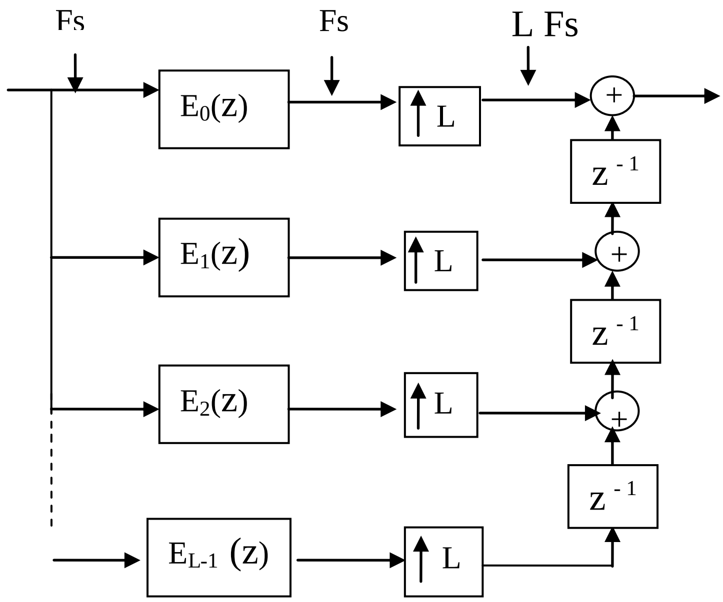


Fig 3.5: Computationally Efficient Interpolator Structure

III.6 Computationally Efficient Fractional Sampling Rate Converter

The following illustrates the implementation of a computationally efficient fractional rate converter with a conversion factor L/M, where L and M are prime. For example, the design of a fractional sample rate converter, which converts the digital signal, received from the ADC from a sampling frequency of 86.625 M samples/sec to 69.3 M samples/sec. This rate change is a ratio of 4/5. The basic form of the desired sampling rate converter is as indicated in figure 2.3. In that structure, the filter $H_1(z)$ operates at 4 x

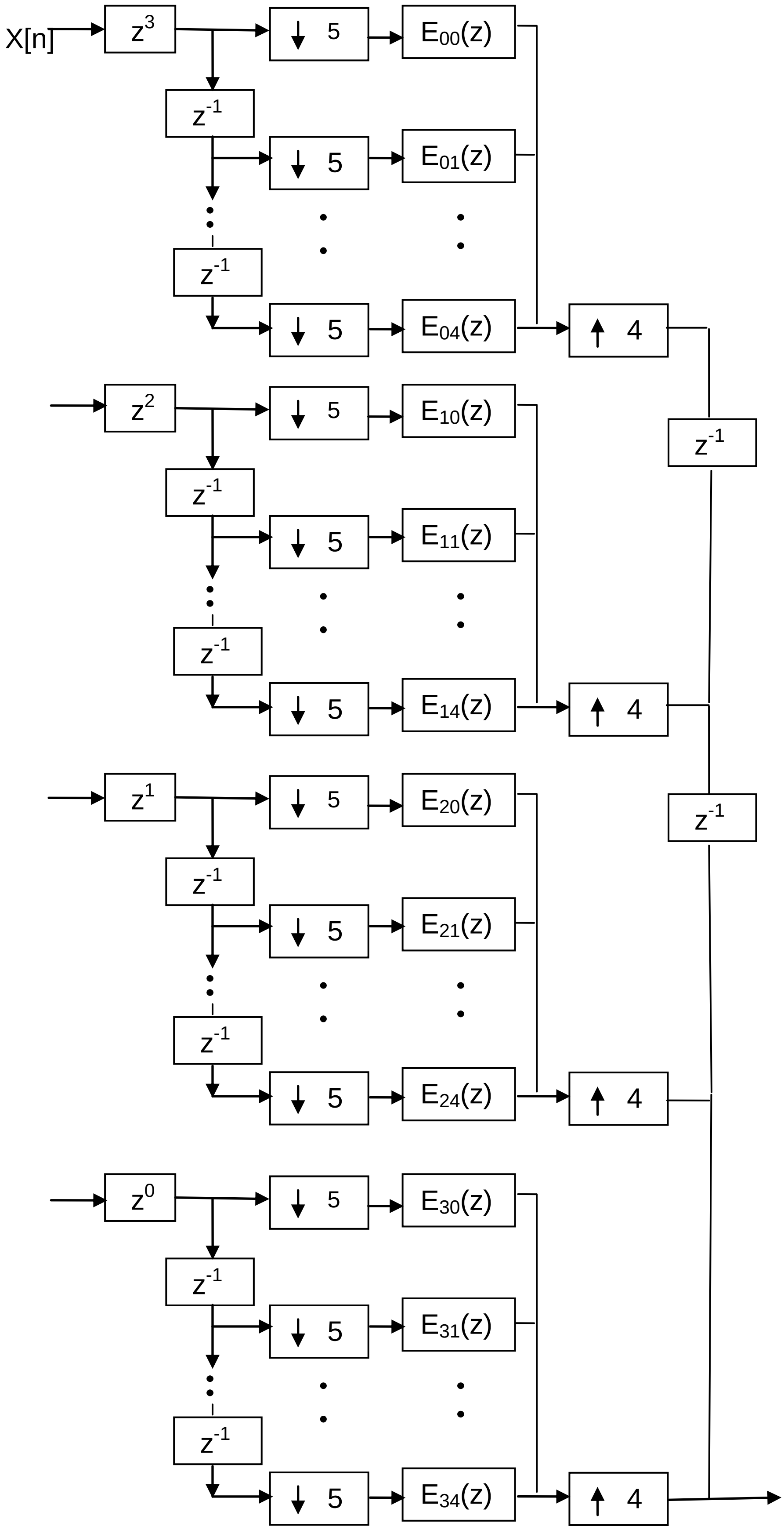


Fig. 3.6: Polyphase Decomposition applied to Fractional Sample Rate Converter

86.625 M samples/Sec = 346.5 M samples/sec. This means that the filter $H_2(z)$ has approximately $1/346.5 \times 10^6$ Sec = 0.003 μsec to produce an output sample to pass to the down sampler. Obviously this filter would be running at a very high speed to keep up with the incoming samples. By realizing the filters $E_o(z)$, $E_1(z)$, $E_2(z)$ and $E_3(z)$ in polyphase form and then applying the second identity, the realization (figure 3.6) in which all filters operate at 17.325 MHz will be obtained.

IV.0 ANALYSIS OF THE RESULT

IV.1 Filter Speed

As mentioned previously, the polyphase decomposition offers substantial computational savings to a multirate system. The filters are placed directly after the down-samplers. In the example, the down-samplers effectively remove four out of every five incoming samples, and only pass every fifth sample along to the filters. Thus, the filters see an incoming sample rate of $^1/_5{}^{th}$ that of the down-samplers, enabling them to run at a rate $^1/_5{}^{th}$ that of the sampling rate of x(n). Table 4.1 compares the required rate of the single filter H(z) from Fig. 2.3 to that of the polyphase filters.

Table 4.1: Comparison of filter speed.

Single filter- H (z)	Polyphase filters
L *Fs = 4* 86.625 MHz = 346.5MHz	Fs/M = 86.625MHz/5 = 17.325MHz

Clearly, operating a filter at 17.325MHz is more desirable than running one at 346.5 MHz. Not only are slower DSP chips less expensive, but the slower speed is also much more efficient in power consumption than the higher rate.

IV.2 Filter Length/Shape

Aside from the slower processing rate required for the polyphase filters, their length is also much shorter than that of the single filter. The Kaiser Window method of filter design is used with the following specifications to design the single low-pass filter:

F_{Pass} = 8.12 MHz(Pass-band)

F_{stop} = 8.663 MHz(Stop-band)

$\delta_1 = 10^{-2}$ (Pass-band ripple)

$\delta_2 = 10^{-4}$ (Stop-band ripple)

F_s = 86 .625 MHz (Sampling frequency)

From the specification given above, the digital frequencies are:

$\omega_p = 2\pi * 8.121/86.625 = 0.1875\pi$ (Pass-band)

$\omega_s = 2\pi*8.663/86.625 = 0.2000\pi$ (Stop-band)
The transition band = $\Delta\omega = \omega_s - \omega_p$
$= 0.2000\pi - 0.1875\pi = 0.0125\pi$
Maximum stop-band gain in dB = $20\log_{10}(0.0001) = -80$dB
Stop-band attenuation in dB = -Maximum stop-band gain in dB = 80dB
The required order of the filter, R, is given by the relation [7];
$R = (A-8)/2.285\Delta\omega = (80-8)/2.285*0.0125\pi = 802$.

Thus the required order of the filter is R = 802. In contrast, after the polyphase decomposition each filter has an order of R = 41. Although the total number of coefficients for the entire polyphase structure is slightly higher than that of the single filter (R_{total} = 20 x 41 = 820), each individual filter only has 41 coefficients and thus requires far less multiply – accumulates and memory fetches per output sample than the single filter.

V.0 CONCLUSION AND RECOMMENDATION FOR FURTHER WORK

V.1 Conclusion

The multirate or sampling rate conversion problem can be viewed as a filter design problem. The filter must satisfy all the specifications that it will avoid signal degradation such as aliasing and imaging while the number of co-efficients must be kept minimal for computational cost issue. A Multirate Digital Signal Processing System such as one that uses only a single filter was shown to be very inefficient. By being smart about designing these filters (using the polyphase decomposition and noble identity combination), one can achieve a much more computationally efficient system, which is of paramount importance in many telecommunication systems that transmit and receive various signals (speech, video, audio, etc.). Multirate Signal Processing has proven to be a powerful method of significantly reducing the number of computation in the sampling rate conversion processes. This is certainly an important issue, especially in the GSM application since resources are scarce in a cellular phone. With the sampling rate reduced, the processor runs at a lower clock rate, thus producing less heat. This will eventually lead to lower power/battery consumption. The consumers will benefit from this feature, since they can have their phones “on” longer without the need to recharge too often.

V.2 Recommendation For Further Work

In many Digital Signal Processing (DSP) application, it is required to change the sampling rate by a non integer ratio. In the case when the SRC factor is a ratio of two large relatively prime integers or irrational numbers, the required order becomes very large, and the overall number of coefficients in polyhase implementation can be impractical. This is an area that requires further studies.

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