Effects of PID Controller Position in Control System Loop on Time Domain Performance Parameters

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ABSTRACT
The effects of proportional-integral-derivative (PID) controller position within the structure of control system loop on system time domain performance parameters were investigated. PID controller was connected as series compensation, feedback compensation and series-feedback compensation respectively. The connected PID controller's gains were tuned. The transfer function for the uncompensated, series compensated, feedback compensated and series-feedback compensated systems were developed, and then subjected to a step input forcing function, which yielded time-domain performance parameters for these systems. The obtained time domain parameters show that the series compensated system has the most superior performance quality compared with other compensated and uncompensated systems.

Keywords: *Feedback Compensated System, PID Controller, Second Order System, Series Compensated System, Series-feedback Compensated System, Time-domain Performance Parameters.*

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I. INTRODUCTION

Controllers are required in all control system loops as the original dynamic of every practical plant does not always result to the desired system response due to the limitation in the plant model parameter adjustment. It should be noted that even though several types of controllers exist in the literature, only the three-term controller called Proportional-Integral-Derivative (PID) controller is considered in this article; due to its industrial popularity, structural simplicity and ease of design methodology[1]. PID controller is the most popular form of feedback control algorithm that deals with important practical issues; as this is evidenced by the fact that over 90% of all control loops in use are of PID type[2]. PID controllers are excellent for reducing the steady-state error and improving the transient response when the system to be controlled is a first-order, second-order or higher-order control system whose model can be approximated by either a first or second-order system model [3].

The three basic PID parameters that guarantee system stability which must be determined are the proportional gain value, K_p, integral gain value, K_i, and the derivative gain value, K_d. In order to obtain satisfactory system response, the afore-mentioned controller gains are often tuned. A lot of works have been reported on the numerous ways by which controller gains can be determined and tuned if required.

To determine controller gain values, [2] first obtained the system stability region as a function of controller gains and then determined the value of the controller gains that gives the required system response within the stability region. An efficient and simple graphical approach to design all stabilizing PID controllers for high-order systems with time delays was presented in [4]. Applying this method can yield all stabilizing PID controller in the K_p, K_i and K_d stability surface and K_i and K_d stability plane for fixed value of K_p. Simple and efficient PID controller design methodology in which the controller parameters are solely determined by plant model was proposed by [1, 5]. The results of using different kinds of performance functions on the three control schemes, on time-domain, frequency-domain and multi objective optimal-tuning of PID control was reported in [6]. The results show significant improvement in system performance. A very simple and effective PID controller tuning rules was used for unstable processes by making use of simple desired closed-loop transfer functions for the direct synthesis method and simple approximations of the plant time delay [7].

The controller subsystem can either be located in the forward or feedback path of the control loop to form series compensation or feedback compensation respectively [3, 8, 9]. It is also possible to have both series compensation and feedback compensation in a single control loop to form series-feedback compensated system. Effects of controller position within the loop of control system on system time domain performance parameters have not been given serious attention as little works have been reported in the literature. [10] connected PID controller in series compensated form to control the firing angle of Static Synchronous Series Compensator (SSSC) that injects variable magnitude sinusoidal voltage in series with the line and almost in phase with the line current into transmission line. Using this method, reactive power compensation and system stability was achieved. In [11] the performance analysis of series connected conventional PID and Fuzzy PID controllers for the control of Continuous Stirred Tank Heater Process was separately considered. Result shows that Fuzzy PID controller has better response and stability compared with the conventional PID controller. Designed PID controller using root locus approach was introduced into the forward path of a simple position control system in [12], the result shows better system response to both step and ramp inputs. The performance of a series connected PID controller was compared with various control algorithms in an automatic drug delivery system in [13], the obtained time domain performance indices shows that PID controller has better performance while Cascaded-lead compensator has the best performance. A novel practical tuning different from the conventional tuning methods for robust series connected PID controller with velocity feed-back for motion control was proposed by [14]. In this work, performance tuning and robustness tuning was carried out separately. The main advantages of this method over others are the simplicity and efficiency. Various tuning methods were employed by [15] to obtain optimum PID controller parameters in a series connected PID controller designed for general aviation aircraft. Out of all the considered methods Zeigler-Nichols method gives the optimal gain values of PID controller parameters. In [16] the performance of a controller connected in series with a DC motor water pump used to control water level in a tank was investigated. The investigation was carried out by configuring the controller as P, PI, PD and PID which was tested via simulation using MATLAB as simulation tool. The results revealed that PID controller configuration achieves super performance. It can be observed that all the aforementioned literatures concentrated on series compensated system. Simple tuning method for two-degree-of-freedom PID controller algorithm for second order processes was presented in [17]. Unlike the proper series-feedback compensation in which the series controller was in cascade with the plant, the feed-forward controller in this case was positioned just between the reference input and the comparator while the feedback controller was positioned along the feedback path. This method gives better result compare to single-degree-of-freedom PID controller for second order processes without time delay. The effects of PID controller in system of mobile satellite dish network when connected as Series and feedback compensated systems were investigated by

[18]. Because of the involvement of higher order plant and time delay due to distributed nature of this system, complex PID controller design methodology was used. Considered in the work were uncompensated, series compensated and feedback compensated systems while the series-feedback compensated system was left out.
The investigation of effects of PID controller position within the loop of a control system on system time domain performance parameters was presented in this article. The expressions for calculating K_p, K_i and K_d proposed by [5] for second order systems was adopted and their determined values were substituted into the PID controller transfer function. The resulting controller was then connected to the system as series compensation, feedback compensation and combination of the two that is series-feedback compensation. After integrating the controller and plant transfer function for each of the controller location structure namely; uncompensated, series compensated, feedback compensated and the series-feedback systems, they were then subjected to a step input forcing function one after the other in other to obtain time domain performance parameters for these systems. The system time domain performance parameters to be determined are the rise time, $\boldsymbol{T_r}$, time to peak overshoot, $\boldsymbol{T_p}$, percentage overshoot, **P.O** and settling time, $\boldsymbol{T_s}$, of the systems dynamic time response.

II. SYSTEM CLOSED-LOOP TRANSFER FUNCTIONS

In this section, the closed-loop transfer function for series compensated, feedback compensated and series-feedback compensated systems were derived in order to determine the time domain performance parameters. The block diagram of series compensated system is as shown in Figure 1. The plant is represented by a linear system with transfer function $G_p(s)$ which determines the output signal $Y(s)$ based on the series control signal$U_f(s)$. The series controller is also linear with transfer function $G_{cf}(s)$ which determines the series control signal $U_f(s)$ based on the error signal $E(s)$. The closed-loop transfer function for series compensated system was determined using the block diagram of Figure 1 and is as shown in equation (1).

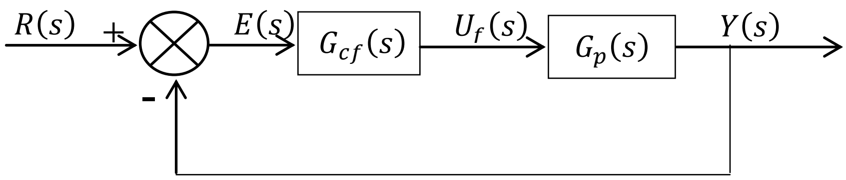

Figure 1: Block Diagram of Series Compensated System

$$\frac{Y(s)}{R(s)} = \frac{G_{cf}(s)G_p(s)}{1+G_{cf}(s)G_p(s)} \quad (1)$$

$Y(s)$ = Actual output
$R(s)$ = Reference input
$G_{cf}(s)$ = Series controller transfer function
$G_p(s)$ = Plant transfer function

A typical feedback compensated system is as illustrated in the block diagram shown in Figure 2. In this case the plant is represented by a linear system with transfer function $G_p(s)$ which determines the output signal $Y(s)$ based on the error signal $E(s)$. The feedback controller is also linear with transfer function $G_{cb}(s)$ which determines the control signal $U_b(s)$ based on system output signal $Y(s)$. The closed-loop transfer function for feedback compensated system obtained from the block diagram of Figure 2 is as shown in equation (2).

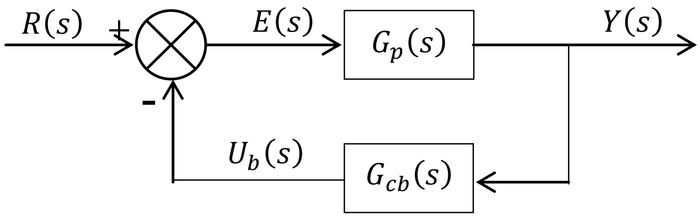

Figure 2: Block Diagram of Feedback Compensated System

$$\frac{Y(s)}{R(s)} = \frac{G_p(s)}{1+G_{cb}(s)G_p(s)} \quad (2)$$

where $G_{cb}(s)$ = Feedback controller transfer function. Series-feedback compensated system is a combination of both the series and feedback compensated systems and is as illustrated in the block diagram of Figure 3. In this case the system is represented by a linear system with transfer function $G_p(s)$ which determines the output signal $Y(s)$ based on the series control signal $U_f(s)$. The series controller is also linear with transfer function $G_{cf}(s)$ which determines the series control signal $U_f(s)$ based on the error signal $E(s)$.The feedback controller is also linear with transfer function $G_{cb}(s)$ which determines the control signal $U_b(s)$ based on system output signal $Y(s)$.

Equation (3) shows the closed-loop transfer function for series-feedback compensated system obtained from the block diagram of Figure 3.

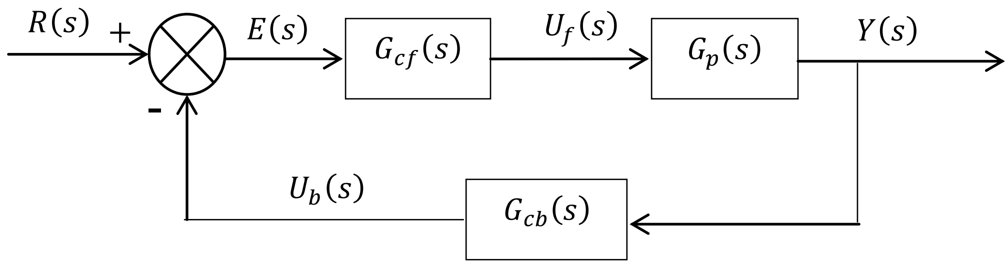

Figure 3**:** Block Diagram of Series-Feedback Compensated System

$$\frac{Y(s)}{R(s)} = \frac{G_{cf}(s)G_p(s)}{1+G_{cf}(s)G_{cb}(s)G_p(s)} \quad (3)$$

III. DESIGN OF PID CONTROLLER

A hypothetical second order plant with transfer function, $G_p(s)$ of the form shown in equation (4) was used in this study. This is because PID controllers are excellent in the control of second order systems and several higher order systems that can easily be approximated by second order system. Moreover, the required expressions for determining the system time domain performance parameters have already been established for second order systems.

$$G_p(s) = \frac{{\omega_n}^2}{s^2+2\xi\omega_n s+{\omega_n}^2} \quad (4)$$

Where:

ξ = plant damping ratio

ω_n = plant natural frequency

PID controller design involves the determination of the controller gains value that will guarantee system stability and also give the desired system performance. The transfer function for PID controller, $G_{pid}(s)$ is as given in equation (5),

$$G_{pid}(s) = \frac{K_d s^2+K_p s+K_i}{s} \quad (5)$$

Where

K_p = Proportinal gain

K_i = Integral gain

K_d = Derivative gain

Further work by [5] on selection of PID controller gains based on system parameters reveals that improved second order system response to step input can be obtained by using controller gains value expressed in equations (6), (7) and (8) respectively.

$$K_p = \frac{4}{\xi\omega_n} \quad (6)$$

$$K_i = \frac{0.15\omega_n}{\xi} \quad (7)$$

$$K_d = \frac{1}{\xi\omega_n} \quad (8)$$

It is obvious from equations (6) to (8) that the controller gains are determined solely by the value of plant natural frequency and damping ratio. To allow for controller tuning, equations (6), (7) and (8) were generalised by rewriting them as shown in equations (9), (10) and (11) respectfully.

$$K_p = \frac{\alpha}{\xi\omega_n} \quad (9)$$

$$K_i = \frac{\beta\omega_n}{\xi} \quad (10)$$

$$K_d = \frac{\gamma}{\xi\omega_n} \quad (11)$$

Where α, β and γ are the proportional, integral and derivative gains tuning factor.

The closed-loop transfer function for uncompensated system was obtained from equation (4), with unity feedback assumed, as shown in equation (12).

$$\frac{Y(s)}{R(s)} = \frac{A_1}{s^2+A_2 s+2A_1} \quad (12)$$

Where:

$A_1 = \omega_n{}^2$

$A_2 = 2\xi\omega_n$

Now that the plant transfer function and the controller transfer functions and structure are known these transfer functions can be substituted into the closed-loop transfer functions of equations (1), (2) and (3) to yield equations (13), (14) and (15) respectively.

$$\frac{Y(s)}{R(s)} = \frac{A_1K_ds^2+A_1K_ps+A_1K_i}{s^3+(A_1K_d+A_2)s^2+(A_1K_p+A_1)s+A_1K_i} \quad (13)$$

$$\frac{Y(s)}{R(s)} = \frac{A_1s}{s^3+(A_1K_d+A_2)s^2+(A_1K_p+A_1)s+A_1K_i} \quad (14)$$

$$\frac{Y(s)}{R(s)} = \frac{A_1K_ds^3+A_1K_ps^2+A_1K_is}{(A_1K_d{}^2+1)s^4+(2A_1K_pK_d+A_2)s^3s+K_i{}^2\ldots} \quad (15)$$

$$+(K_p{}^2+K_iK_d+A_1)s^2+2K_pK_i.$$

Where K_p, K_i and K_d are as defined in equations (9), (10) and (11) respectively.

IV. SYSTEM RESPONSE SIMULATION

The step input was used as the test signal in this work as it is capable of handling the worst scenario. The system quality of performance (QoP) is based on the value of system closed-loop time domain performance parameters namely rise time $(\boldsymbol{T_r})$ peak time, $(\boldsymbol{T_p})$ percentage overshoot $(\boldsymbol{P.O})$ and settling time $(\boldsymbol{T_s})$. A hypothetical system parameters of $\xi = 0.5$ and $\omega_n = 4$ radian/sec were used in the simulation.

After substituting the value of ξ and ω_n in equations (12) to (15), the closed-loop step response for the uncompensated, series compensated, feedback compensated and series-feedback systems were simulated using Matlab software. PID gains tuning factor of 4, 0.15 and 1 respectively which correspond to that used by [5] was adopted as the reference. The controller was tuned by adjusting the tuning factors on a trial and error basis in order to give the best response for any of these systems whose response is closer to reference output. The values of controller gain under this condition are the gain required for the determination of the system time domain performance parameters. For easy of comparison the step responses for these systems were plotted on the same graph as shown in the response graph of Figure 4 for system with un-tuned controller gains and Figure 5 for system with tuned controller gains.

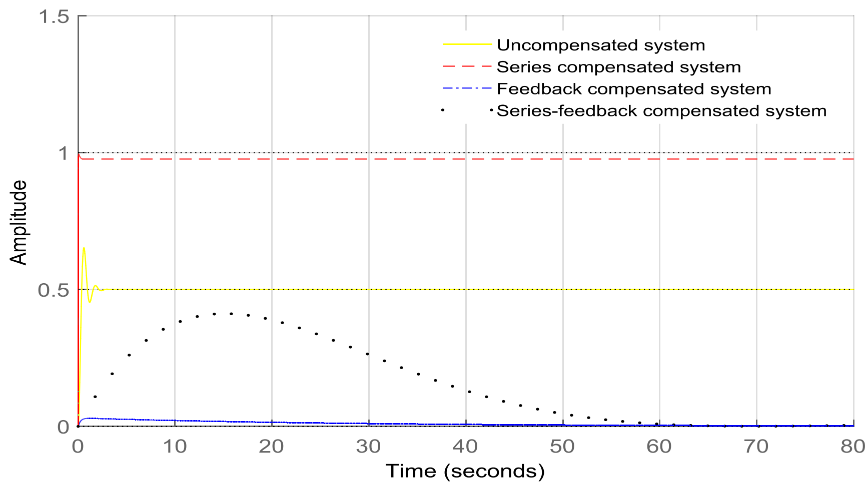

Figure 4: System Step Response before Tuning the Controller

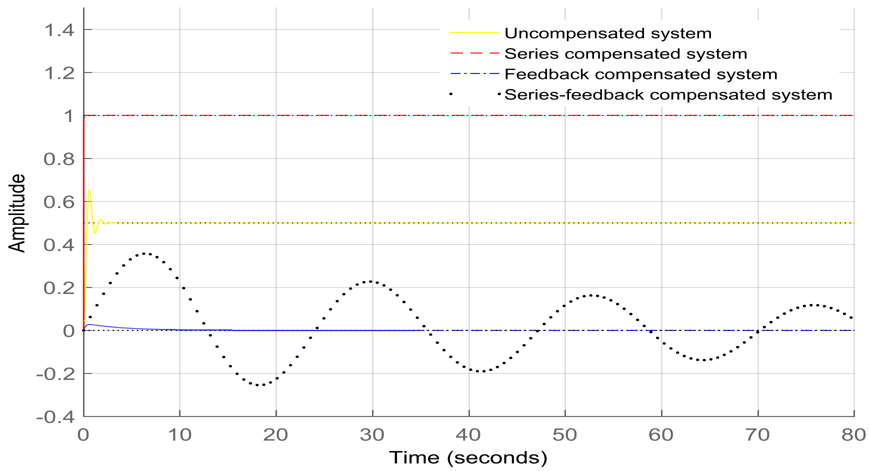

Figure 5: System Step Response after Tuning the Controller

V. RESULTS AND DISCUSSION

The time domain performance parameters extracted from the simulation graphs of Figures 4 and 5, and the controller gains tuning factor values that give these responses are as presented in Tables 1 and 2 respectively.

Table 1: System time domain performance parameters before tuning and controller gain tuning factor values

Controller gains tuning factor before tuning, $\alpha = 4$, $\beta = 0.15$ and $\gamma = 1$					
System category	T_r **(sec)**	T_p **(sec)**	$P.O$ **(%)**	T_s **(sec)**	**Remarks**
uncompensated system	0.247	0.599	30.5	1.94	The system settle to 0.5
Series Compensated System	0.0172	Not applicable	Not applicable	Not applicable	The system does not get to reference input (unity step)
Feedback Compensated System	0	1.17	∞	108	The system settle to 0
Series-feedback Compensated System	0	15.8	∞	121	The system settle to 0

Table 2: System time domain performance parameters after tuning and controller gain tuning factor values

Controller gains tuning factor after tuning, $\alpha = 4$, $\beta = 1$ and $\gamma = 0.8$					
System category	T_r **(sec)**	T_p **(sec)**	$P.O$ **(%)**	T_s **(sec)**	**Remarks**
Uncompensated system	0.247	0.599	30.5	1.94	The system settles to 0.5
Series compensated System	0.021	0.0642	0.0584	0.0359	The system settles to reference input (unity step)
Feedback compensated System	0	0.633	∞	16.2	The system settle to 0
Series-feedback compensated System	0	6.23	∞	273	The system settles to 0

As can be seen from the graph of Figure 4, the response of the series compensated system is closer to the reference input than any other system response, therefore the target is to tune the controller so as to make this system response track the reference input. Since this system response can never get to the reference input under this condition, $\boldsymbol{T_p}$, $\boldsymbol{P.O}$, and $\boldsymbol{T_s}$ cannot be determined for series compensated system. Figure 4 also shows that there is improvement on the performance of the series compensated system over the uncompensated and other compensated systems. The performance of the feedback and series-feedback compensated systems are degenerated version of the uncompensated system as the two systems settled to zero as time tend to infinity which make the value of percentage overshoot to be infinity. When the controller was tuned, the performance of the series compensated system was improved in terms of $\boldsymbol{T_p}$, $\boldsymbol{P.O}$, and $\boldsymbol{T_s}$ compared to other systems as can be seen from Figure 5 and the values of time domain performance parameters in Table 2. The tuning of the controller gains has no significant improvement on the feedback and series-feedback compensated systems as these systems only track zero instead of the reference input of unity. It should be noted that all the feedback and series-feedback compensated systems time domain performance parameters values in Tables 1 and 2 were obtained based on the fact that these systems track zero level as time tend to infinity.

VI. CONCLUSIONS

PID controller was tuned and the controller parameters were solely determined by plant parameters. The composite system closed-loop transfer function for uncompensated, series compensated, feedback compensated and series-feedback compensated systems were also determined. The results obtained for time domain performance parameters for these systems shows that the series compensated system has superior quality of performance. The performance of feedback compensated and series-feedback compensated systems are worse than those of uncompensated and series compensated systems as these system responses cannot track the unity step input instead they tend to zero as the time tend to infinity.

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